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Math 202 - Bonus Problem 2

Due on November 11 (with your regular homework).

This problem is entirely optional, and will not hurt your grade at all (if you don't even try to solve it).

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$e$  is irrational.

Define the number  $e$  by the power series

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}, \quad n! = n(n-1)\cdots 1$$

The point of this problem is to show that  $e$  is irrational.

1. Show that  $2 < e < 3$ . So  $e$  is definitely not an integer.
2. By contradiction, say  $e = \frac{p}{q}$ , where  $p$  and  $q$  are positive integers with  $q \geq 2$ . Show that

$$e q! = N + \frac{c}{q+1},$$

where  $N$  is an integer and  $0 < c < e$ . Conclude that  $\frac{c}{q+1}$  must be an integer.

3. Then show that this contradicts  $e < 3$  and  $q+1 \geq 3$ .

Try as many parts as you can.