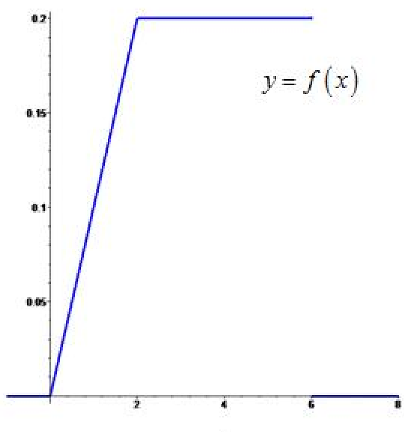
MATH 104, Fall 2009, Practice Exam II

Multiple choice questions

The worth of each question is one less than the number of possible answers, for example, if the choices are (a), (b), (c), (d), and (e), then a correct answer is worth 4 points. An incorrect answer is worth -1. No calculators are allowed on this exam.

1. The function f whose graph is shown below is defined by $f(x) = x/10$ for $0 \leq x \leq 2$, $f(x) = 1/5$ for $2 \leq x \leq 6$, and $f(x) = 0$ everywhere else. Compute the mean of this probability density.



(a) 1
(b) 3
(c) $\frac{10}{3}$
(d) $\frac{17}{5}$
(e) $\frac{52}{15}$
(f) 4

2. Find the arc length of the curve $y = \ln(\cos(x))$, $0 \leq x \leq \pi/3$.

   (a) $\ln(\sqrt{2} - 1)$
   (b) $\ln(\sqrt{3} - 1)$
   (c) $\ln(1 + \sqrt{2})$
   (d) $\ln(1 + \sqrt{3})$
   (e) $\ln(2 + \sqrt{2})$
   (f) $\ln(2 + \sqrt{3})$

3. Find the y-coordinate of the centroid of the region bounded by the curve $y = x^2$ and the line $y = 1$.

   (a) 0.80
   (b) 0.75
   (c) 0.70
   (d) 0.60
   (e) 0.50
   (f) $\sqrt{\pi} - 1$

4. Find the arclength of the curve $3y = 4x$ between the points $(3, 4)$ and $(9, 12)$.

(a) 8

(b) 9

(c) 10

(d) 12

(e) 14

(f) 15

5. Find the area of the surface obtained by rotating the curve $x = \frac{1}{3}\left(y^2 + 2\right)^{3/2}$, $1 \le y \le 3$, about the x-axis.

(a) 16π

(b) $64\pi/3$

(c) 48π

(d) $16\pi^2$

(e) $16\pi + 16\pi^2/3$

6. A wire of length 28 cm is bent into some kind of curve in the half-plane $x > 0$. It is a monotone curve (moving along the curve so that the x-coordinate increases, also increases the y-coordinate). Rotating this about the y-axis produces a surface with area $300\pi\,\text{cm}^2$. If instead, it is rotated around the line $x = -5\,\text{cm}$, what will be the resulting surface area in square centimeters?

   (a) 325π

   (b) $300\pi + 25\pi^2$

   (c) 580π

   (d) 580π

7. Find the limit of the sequence $a_n = (n + e^n)^{1/n}$.

   (a) 1

   (b) 2

   (c) e

   (d) $1 + e$

   (e) divergent

8. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{3}{4}$

(d) 1

(e) $\frac{3}{2}$

(f) divergent

9. Find the values of x for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}\left(2x+\frac{1}{2}\right)^n$ converges.

(a) $x = -\frac{1}{4}$ only

(b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(d) $\left[-\frac{3}{4}, \frac{1}{4}\right)$

(e) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

(f) $\left[-\frac{3}{2}, \frac{1}{2}\right)$

(g) all real values of x

10. How many terms of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ must we sum to be sure the partial sum s_n is within 0.0001 of the sum S?

(a) 100
(b) 300
(c) 1,000
(d) 10,000
(e) 1,000,000

11. Which of the following series will, when rearranged, sum to different values?

(1) $\sum_{n=1}^{\infty} \frac{1}{n}$

(2) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

(3) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

(a) none
(b) 1
(c) 2
(d) 3
(e) 2 and 3
(f) all of the above

Free answer questions

Each of these questions is worth 8 points. Please **show all work** that you wish to be considered for partial credit and **put a box** around the final answer to each (part of the) problem.

12. A boat has a cross-section shaped like a V. The height, from the bottom to the top is 7 meters. Water begins to leak into the boat. The boat will be unstable once the center of gravity of the water is higher than half the height of the boat. How many meters high must the water be inside the boat in order for this to occur?

13. Estimate $e^{-1/2}$ to three decimal places by using the power series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

14. Suppose that W and X are random variables and that for all t,

$$\mathrm{Prob}\,(W \le t) = [\mathrm{Prob}\,(X \le t)]^2\ .$$

If X has density $2x$ on $[0,1]$, find the density of W.

15. There is a positive, increasing, differentiable function f with $f(0) = 1$ such that the arclength of the curve $y = f(x)$ between $(0,1)$ and $(x, f(x))$ is equal to $\sinh(x)$ for all positive x. What is f?