Supplementary exam problems for Chapter 12

1. Let $b > a > 1$ be integers. Compute the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+a)(n+b)} .$$

Hint: use a partial fraction expansion.

Extra problem: Can you sum $1/[(n+a)(n+b)(n+c)]$ the same way?

2. Define p_n and q_n recursively by $p_n = q_n = 1$ and

$$\begin{aligned} p_{n+1} &= 2q_n + p_n \\ q_{n+1} &= 3q_n + p_n . \end{aligned}$$

Let $a_n = p_n/q_n$. Prove that a_n converges to a finite limit and evaluate the limit.

3. A superball, when dropped from height h, returns to height $h/(1+h)$. Does such a ball travel a finite distance or an infinite distance if dropped from a positive height and left to bounce forever?

4. Write the series $\frac{1}{1} + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \cdots$ in $\sum$ notation. Then determine whether the series converges.